

A Counterexample in Number Theory: Falsification of a Computational Conjecture

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Abstract

We report the falsification of the following conjecture: For every even integer $n > 10,000$, there exists a Goldbach partition $n = p + q$ (where p and q are primes) such that both p and q are 'isolated' within a window of size $W(n) = \text{floor}(0.8 * \ln(n) * \ln(\ln(n)))$. Specifically, there are no other primes in . A counterexample was discovered computationally: witness = 50000. This result was obtained by the SOVEREIGN autonomous research system.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Goldbach conjecture — extend computational verification. The result was obtained autonomously by the SOVEREIGN Research Kernel, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by the SOVEREIGN Research Kernel and subjected to automated falsification search:

Conjecture 1. *For every even integer $n > 10,000$, there exists a Goldbach partition $n = p + q$ (where p and q are primes) such that both p and q are 'isolated' within a window of size $W(n) = \text{floor}(0.8 * \ln(n) * \ln(\ln(n)))$. Specifically, there are no other primes in the interval $(p - W(n), p + W(n))$ other than p itself, and similarly for q . This suggests that for large n , Goldbach partitions can be found using primes that are surprisingly far from their nearest prime neighbors relative to the average prime gap.*

3 Counterexample

Theorem 1 (Falsification). *The conjecture above is **false**. A counterexample is given by:*

$$witness = 50000$$

Proof. Direct computation verifies that the witness 50000 satisfies the negation of the conjecture. The verification was performed by the SOVEREIGN counterexample search module. \square

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the SOVEREIGN system refine its mathematical intuitions.