

# A Counterexample in Number Theory: Falsification of a Computational Conjecture

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## Abstract

We report the falsification of the following conjecture: For every integer  $N \geq 2$ , let  $P_N$  be the set of primes of the form  $k^2+1$  less than or equal to  $N$ . Let  $M_N$  be the maximum gap between consecutive elements in the sorted sequence  $P_N$  (defining the first gap as  $p_1 - 0$ ). Then  $M_N$  is strictly less than 4. A counterexample was discovered computationally: witness = {'N': 50000, 'max\_gap': 7760, 'bound': 2129.9705579159213, 'primes': [2, 5, 17, 37, 101, 197, 257, 401, 577, 677, 1297, 1601, 2917, 3137, 4357, 5477, 7057, 8101, 8837, 12101, 13457, 14401, 15377, 15877, 16901, 17957, 21317, 22501, 24337, 25601, 28901, 30977, 32401, 33857, 41617, 42437, 44101]}. This result was obtained by Assignee Research.

## 1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Primes of form  $n^2+1$  — density conjecture. The result was obtained autonomously by Assignee Research, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

## 2 The Conjecture

The following conjecture was generated by Assignee Research and subjected to automated falsification search:

**Conjecture 1.** *For every integer  $N \geq 2$ , let  $P_N$  be the set of primes of the form  $k^2+1$  less than or equal to  $N$ . Let  $M_N$  be the maximum gap between consecutive elements in the sorted sequence  $P_N$  (defining the first gap as  $p_1 - 0$ ). Then  $M_N$  is strictly less than  $4 * \sqrt{N} * \ln(\ln(N+2))$ .*

### 3 Counterexample

**Theorem 1** (Falsification). *The conjecture above is **false**. A counterexample is given by:*

*witness = {'N' : 50000, 'max\_gap' : 7760, 'bound' : 2129.9705579159213, 'primes' : [2, 5, 17, 37, 101, 197, 257, 401, 577, 677, 1297, 1601, 2917]}*

*Proof.* Direct computation verifies that the witness {'N' : 50000, 'max\_gap' : 7760, 'bound' : 2129.9705579159213, 'primes' : [2, 5, 17, 37, 101, 197, 257, 401, 577, 677, 1297, 1601, 2917]} satisfies the negation of the conjecture. The verification was performed by the Assignee Research counterexample search module.  $\square$

### 4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the Assignee Research system refine its mathematical intuitions.