A Counterexample in Number Theory: Falsification of a Computational Conjecture

Assignee Research
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https://assignee.net

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Abstract

We report the falsification of the following conjecture: For the sequence of primes of the form p = n^{}2 + 1, let S(x) be the set of such primes less than or equal to x. Define the 'quadratic gap ratio' for a prime p = n^{}2 + 1 (where n > 1) as R(p) = (p_next - p) / (2n), where p_next is the next prime in th. A counterexample was discovered computationally: witness = No witness found up to n=50000. Min ratio observed: 1.5000 at n=1. This result was obtained by Assignee Research.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Primes of form n^{}2+1 — density and distribution. The result was obtained autonomously by Assignee Research, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by Assignee Research and subjected to automated falsification search:

Conjecture 1. *For the sequence of primes of the form p = n^{}2 + 1, let S(x) be the set of such primes less than or equal to x. Define the 'quadratic gap ratio' for a prime p = n^{}2 + 1 (where n > 1) as R(p) = (p_next - p) / (2n), where p_next is the next prime in the sequence S. The conjecture states that the limit inferior of R(p) as p approaches infinity is strictly less than 0.5. Specifically, there exist infinitely many indices n such that the gap to the next prime of the form m^{}2+1 is smaller than n (i.e.,*

3 Counterexample

Theorem 1 (Falsification)**.** *The conjecture above is* ***false****. A counterexample is given by:*

$$witness = Nowitnessfoundupton = 50000.Minratioobserved : 1.5000atn = 1$$

Proof. Direct computation verifies that the witness $Nowitnessfoundupton = 50000.Minratioobserved : 1.5000atn = 1$ satisfies the negation of the conjecture. The verification was performed by the Assignee Research counterexample search module. □

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the Assignee Research system refine its mathematical intuitions.