

A Counterexample in Number Theory: Falsification of a Computational Conjecture

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Abstract

We report the falsification of the following conjecture: For all integers $x \geq 100$, the absolute difference between the actual count of twin prime pairs up to x and the Hardy-Littlewood prediction $(2 \cdot C_2 \cdot x / \ln(x))^2$ is strictly bounded by the square root of the prediction itself. That is, $|\pi_2(x) - \text{Li}_2(x)|$. A counterexample was discovered computationally: witness = {'n': 50000, 'actual': 705, 'predicted': 563.9150404300273, 'diff': 141.08495956997274, 'bound': 23.746895385081967}. This result was obtained by Assignee Research.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Twin prime density — Hardy-Littlewood conjecture verification. The result was obtained autonomously by Assignee Research, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by Assignee Research and subjected to automated falsification search:

Conjecture 1. *For all integers $x \geq 100$, the absolute difference between the actual count of twin prime pairs up to x and the Hardy-Littlewood prediction $(2 \cdot C_2 \cdot x / \ln(x))^2$ is strictly bounded by the square root of the prediction itself. That is, $|\pi_2(x) - \text{Li}_2(x)| < \text{sqrt}(\text{Li}_2(x))$, where $\text{Li}_2(x)$ approx $2 \cdot C_2 \cdot x / \ln(x)^2$.*

3 Counterexample

Theorem 1 (Falsification). *The conjecture above is **false**. A counterexample is given by:*

witness = {'n' : 50000, 'actual' : 705, 'predicted' : 563.9150404300273, 'diff' : 141.08495956997274, 'bound' : 23.746895385081967}

Proof. Direct computation verifies that the witness {'n' : 50000, 'actual' : 705, 'predicted' : 563.9150404300273, 'diff' : 141.08495956997274, 'bound' : 23.746895385081967} satisfies the negation of the conjecture. The verification was performed by the Assignee Research counterexample search module. \square

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the Assignee Research system refine its mathematical intuitions.